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Evaluating the Outcome of the 2022 United States Redistricting Cycle: A Nonpartisan Review

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Evaluating the Outcome of the 2022 United States Redistricting Cycle: A Nonpartisan Review

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May 12, 2022

Contents

1	Introduction	1
2	Mathematically Quantifying Fair Redistricting	2
2.1	Geometric Measures	2
2.2	Political Measures	3
2.3	Ensemble-Based Measures	4
2.4	Jurisdictional Partisan Advantage	4
2.5	Legal Context and Requirements for Redistricting.....	5
2.5.1	<i>Adams v. DeWine</i>	6
2.5.2	<i>Harper v. Hall</i>	7
2.5.3	<i>Szeliga v. Lamone</i>	8
2.5.4	<i>Harkenrider v. Hochul</i>	8
3	Results	8
3.1	Methodology.....	8
3.2	Jurisdictional Partisan Advantage	11
3.3	Comparison to Other Measures	13
3.3.1	California.....	13
3.3.2	Connecticut	13
3.3.3	Florida.....	14
3.3.4	Georgia.....	14
3.3.5	Illinois	14
3.3.6	Massachusetts	15
3.3.7	Nevada.....	15
3.3.8	New Jersey.....	15
3.3.9	Oklahoma.....	15
3.3.10	South Carolina	15
3.3.11	Texas.....	16
3.3.12	Wisconsin.....	16
3.4	A Comparison of Map Authors.....	16
4	Conclusion	18
A	Comparison of Measures Table	18

I Introduction

Every decade the decennial US census is followed by a tumultuous process of redrawing state voting maps. This process gives states an opportunity to update their voting districts to account for shifting population distributions. In many states, this occurs at three levels: the US congressional districts

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determined via apportionment to the US House of Representatives, State Senate districts, and State House districts. However, those entrusted with redrawing the maps also wield the power to architect election outcomes for the decade to come. Partisan actors can extrapolate available geographic voting data, polling results, and demographic information to effectively simulate future election outcomes on a variety of districting proposals. Such a manipulation of the geographic boundaries of voting districts is colloquially known as *gerrymandering*.

While gerrymandering has occurred during the redistricting process since the very beginning of the country's history, its impact is nowadays far more pronounced. The rapid development of technology and large-scale data processing capacities facilitates effective modeling at the level of census-blocks. Moreover, redistricting has become part of the strategem of both the Democratic and Republican political parties. When political parties control this process and exploit it to choose the voters that elect them, they can provide disproportionate advantage to their party, ensure re-election of incumbents, eliminate competitive elections, disenfranchise particular demographic voting blocs, and generally undermine democratic processes. In this paper, we focus on the implications of gerrymandering to partisan fairness.

To ensure fair democratic representation, it is critical to mathematically quantify when gerrymandering has occurred so that the unfair districting plans can be disputed in court. We discuss the nuances of several proposed measures in the following section.

In this paper, we compute the *Jurisdictional Partisan Advantage* (introduced in Eguia 2021) of the enacted US Congressional districting plans in each state involved in the process. This benchmark is designed to discount inherent geographical partisan advantage and reveal the residual advantage. The results we use are those computed as of May 10th, 2022, recently after Florida has enacted its first plan, New York's plan has been ruled a gerrymander by the courts, and Kansas' map has been thrown out by the courts. We find that, throughout the country as a whole, the Republican party has an advantage of 17.88 seats thus far in the 2022 redistricting cycle. We analyze the state-specific outcomes in detail in Section 3.2. The aggregate has the potential to shift from the three states remaining without maps, Kansas, Missouri, New Hampshire, and New York. For example, before the New York map was thrown out it yielded some Democratic advantage (0.84 seats). The Kansas map yielded very mild Republican advantage (0.47 seats). The effects of Missouri and New Hampshire are not expected to be especially significant on the aggregate outcome.

In recent years, as public awareness of gerrymandering has grown, many states have enacted processes to facilitate fairer redistricting. The 2022 redistricting cycle is the first with a significant number of redistricting maps drawn by nonpartisan groups. In Section 3.4, we compare the computed Jurisdictional Partisan Advantage in state's whose maps were drawn by different actors. These include independent commissions, split state legislatures, unified state legislatures, and the courts. We find that the courts and independent commissions most consistently created US Congressional districting plans with minimal Jurisdictional Partisan Advantage.

2 Mathematically Quantifying Fair Redistricting

Those attempting to quantify the fairness of redistricting maps have to weigh a multitude of factors. Effective quantitative measures must have some well-defined mathematical basis. However, given that the primary purpose of these measures is to substantiate arguments in court, they must also be accessible to lawyers and judges unwilling to base their judgments solely on expert opinion. See, for example, the dissent in *Harkenrider v. Hochul* as described in Section 2.5.4.¹ Some of the most rigorous metrics are therefore limited by their complexity and explicability. As we will see, Jurisdictional Partisan Advantage enjoys a balance of simplicity and theoretical legitimacy.

2.1 Geometric Measures

Popular discussions of gerrymandering invariably describe it as the drawing of districts so strangely shaped that they must have an iniquitous purpose. Highly publicized distorted districts like Pennsylvania's 7th Congressional District from the last redistricting cycle, known to some as "Goofy Kicking Donald Duck," perpetuate this narrative. Initial efforts to mathematically quantify gerrymandering attempted to formalize the notion of an oddly shaped district. Some of the earliest measures include

¹*Harkenrider v Hochul*, Sup. Ct. Steuben County Index No. E2022-0116cv Doc# 243 (2022), 7.

the Reock Score (Reock 1961) and Schwartzberg compactness score (Schwartzberg 1966). Later geometric measures include the widely used Polsby-Popper compactness measure (Polsby – Popper 1991). Since the introduction of these measures, 18 states have adopted compactness requirements for districting plans in their constitutions.

The Reock Score is the ratio of the area of the minimum bounding circle enclosing the geometry of a district to the area of that district. A related measure is the ratio of the area of the convex hull of the geometry of a district (minimum convex polygon bounding the district) to the area of the district. The motivation for the measures is that fair districts tend toward more circular or more convex shapes.

Schwartzberg compactness and Polsby-Popper compactness are highly related measures. The Schwartzberg compactness of a district is the ratio of the perimeter of a district to the circumference of a circle of equal area. The Polsby-Popper compactness of a district is instead the ratio of the area of the district to the area of a circle with circumference equal to the perimeter of the district.

A clear boon of these measures is their simplicity. Moreover, by imposing compactness requirements in redistricting, thoughtless gerrymanders can indeed be prevented.

However, these measures fail when applied as a measure of partisan fairness rather than a measure of the geometry of a district. They ignore the fact that gerrymandering requires deriving advantage across a districting plan as a whole, not a singular district. Measures of singular districts are inherently weak.

Such geometric criteria are also easily gameable. Note that, in reality, many districts have fractal boundaries (see Maryland’s coastline, for example). Defining the perimeter of a district then requires fixing a resolution at which to measure perimeter. But reasonable resolutions depend on context and drastically affect the resulting scores of these measures (see Barnes – Solomon 2020 for a more complete critique of compactness measures). As such, setting legal bounds for geometric measures of fair districts is ineffective on its own.

Most importantly, while infamous gerrymanders might include unshapely districts, including such districts is neither necessary nor sufficient for unfair redistricting. Unshapely districts may be fair districts from the perspective of yielding average outcomes and may be necessary in states with fractal boundaries. Conversely, districting plans including convex districts encompassing large portions of city centers may easily represent unfair packings of voters without any visual indication of manipulation. As such, geometric measures are not reliable indicators of manipulation in the redistricting process.

2.2 Political Measures

Given the weakness of geometric measures, numerous other tools have been introduced. Partisan symmetry is the idea that a certain percentage of the vote should translate to the same number of seats, regardless of the party achieving that vote percentage.

A common tool in partisan symmetry analyses is the “seats-votes” curve. This is the graph of the number of seats a party receives versus the percent of the vote they receive. The principle of partisan symmetry translates to each party having roughly the same seats-votes curve for a fair map (see Gary – Robert 1987). While in theory this seems like a simple and effective principle, many authors have demonstrated that partisan symmetry does not always appear organically. For example, maps implying high partisan symmetry in Utah elections are enormous outliers in large ensembles of randomly generated redistricting plans (see DeFord et al. 2021a). Moreover, generating seats-votes curves to check partisan symmetry requires an unsubstantiated extrapolation of available voting data.

Another popular political measure is the efficiency gap (Stephanopoulos – McGhee 2014). In the context of the efficiency gap, a “wasted vote” is defined as a vote cast toward a candidate that does not contribute toward their election victory. That is, if the candidate a person voted for loses the election, the vote is wasted. All of the “surplus votes” for a candidate beyond the number required to win them the election are also wasted. Then, the efficiency gap is the difference in wasted votes

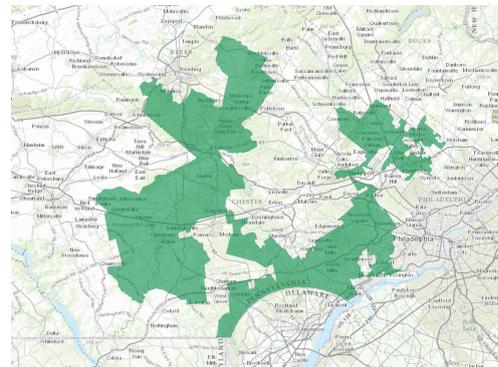


Figure 1: Pennsylvania’s 7th Congressional District

for each party over the number of total votes cast. As with partisan symmetry, the appeal of the efficiency gap is both its simplicity and its seemingly clever encapsulation of partisan fairness. However, achieving a low efficiency gap is often unachievable. Massachusetts is a classic example—the distribution of Republican voters across the state leads to an almost guaranteed Democratic sweep, rendering every Republican vote wasted regardless of the districting plan. Any state in which every district votes for the majority party has a terrible efficiency gap score (this is a key factor in the dissenting argument of *Adams v. DeWine*² as discussed in Section 2.5.1).

Several related measures have been deemed more reliable alternatives to the efficiency gap while still encapsulating the idea of partisan fairness. One of the most popular is the efficiency gap with a 2:1 ideal seat margin to vote margin ratio. For ease of notation, we henceforth refer to this measure as PA|DB, pronounced “Partisan Advantage given DB,” with DB for “Double Bonus” as in Ramsay 2021. Other related measures have the minority party getting twice the square of their vote percentage of the seats, $PA|_2^2$, and the same again with the ratio of the seats being the cube of the ratio of the votes, $PA|_3^3$. $PA|_2^2$ is described further in Pegden et al. 2017; Barton 2022. $PA|_3^3$ is introduced in Tufté 1973.

In particular, if d is the Democratic vote percentage and Democrats win a minority of the vote, for $PA|_2^2$ then Democrats are assigned $2d^2$ percent of the vote (and Republicans the remaining vote). Similarly, for $PA|_3^3$, then Democrats are assigned $d^3/(3d^2 - 3d + 1)$ percent of the total seats.

We compare our results to these three alternatives to the original efficiency gap in Section 2.5.

2.3 Ensemble-Based Measures

It may feel as though every metric of gerrymandering is doomed for failure. Fortunately, recently developed tools, applications of widely used Markov Chain Monte Carlo (MCMC) methods, appear to be much more reliable methods of identifying gerrymandering. These methods assume that, if you could simulate the same election on all possible legal districting plans and collect the partisan outcomes, then a fair plan’s partisan outcome should be reasonably close to the average outcome. Namely, enacted plans should not yield outliers in election results. This is a true necessary condition for a fair districting plan. It is notably still not sufficient, given that partisan manipulation comes in many forms.

New algorithms such as Recombination, Merge-Split, and others (DeFord et al. 2021b, Eric et al. 2020, Cory – Kosuke 2021) randomly sample from the space of all legal redistricting plans. This, in theory, allows for a finite *ensemble* of maps to model the behavior of the entire space of maps as a collective. There is difficulty in proving these samples are uniform samples and still work to be done in developing such rigorous justifications.

Using pre-existing election data, it is then possible to simulate elections on these generated ensembles and compare those results to simulated results on the proposals in question.

In many ways these measures are the best existing measures for quantifying gerrymandering. One notable flaw however is their complexity. It is critical to be able to effectively justify claims that a proposal is unfair in court, but the proofs of the legitimacy of these algorithms are out of reach for most non-mathematicians.

An additional flaw arises in determining whether the algorithm generating the maps has “converged” to providing a random sample of the maps. This becomes an especially challenging issue when the algorithm is adjusted to account for other redistricting requirements, such as communities of interest, maintaining existing jurisdictions, and satisfying the Voting Rights Act. This is in part the basis for the dissenting opinion of the New York case *Harkenrider v. Hochul*³ discussed in Section 2.5.

2.4 Jurisdictional Partisan Advantage

In this paper, we compute the Jurisdictional Partisan Advantage, a measure of redistricting fairness introduced in Eguia 2021. The procedure for computing this advantage is as follows. Fix a representative body in a state with k seats. Also fix a voting profile v , an indication of how every citizen voted in some election, and a redistricting map m , an assignment of voting precincts to voting districts corresponding to the representative body. Then, the partisan advantage for a party p with respect to the voting profile, representative body, and map is the difference between the number of seats they

²*Adams v. DeWine*, Ohio St.3d, 2022-Ohio-89, N.E.3d, ¶143.

³*Harkenrider v Hochul*, Sup. Ct. Steuben County Index No. E2022-0116cv Doc# 243 (2022), 9.

win simulating the election on the map m , $s_p(v, m)$, and the outcome according to some benchmark, $s_p(v)$ (notation from Eguia 2021):

$$s_p(v, m) - s_p(v).$$

In this case, the Jurisdictional Partisan Advantage is derived from a carefully chosen $s_p(m)$ based on maps of jurisdictions approximately equal in population to the representative body's districts.

The jurisdictional maps are generated algorithmically. For each state, a jurisdiction population threshold is set of twice the average population of a US Congressional district. Then, each county with more than that population is recursively split, repeatedly making the largest city contained in that county into its own jurisdiction until the remainder of the county is either below the population threshold or there are no remaining cities to make into independent jurisdictions. At that point, the remainder of the county becomes a jurisdiction. This case is quite rare, with only 20 total counties in the country being recursively split in our computations.

Then, the voting profile is simulated on the jurisdictional map. Let T be the total population of state containing the jurisdictional map, k the total number of districts in the map m , and Q the total population of the jurisdictions in which party p wins the largest share of the vote. Then,

$$s_p(v) = k \cdot Q/T.$$

This benchmark amounts to simulating the election on pre-existing jurisdictional boundaries of approximately the size of voting districts and then using that result as a fractional approximation of a fair election outcome.

Why is this jurisdictional benchmark a reasonable object of interest? This benchmark is extremely simple, making it accessible in court. Indeed, it was used in the second affidavit of Professor Moon Duchin in *Harper v. Hall* discussed in the following section⁴. Moreover, it makes no undesirable assumptions about partisan outcomes having some kind of symmetry or neutrality. Rather, it incorporates the “natural asymmetries that arise due to political geography” (Eguia 2021). In this way, this benchmark avoids the issue of needing some unprovable external legitimacy and instead compares numerical outcomes of districting plans and jurisdictional maps. This makes Jurisdictional Partisan Advantage more akin to a simplified version of the ensemble methods than methods such as the efficiency gap.

Of course, this measure has some flaws in its simplicity. Counties are not a perfect approximation of voting districts—many have much smaller populations than a single voting district. Nevertheless, many states emphasize preserving jurisdictional boundaries, whether that be explicit, as in the Michigan Constitution, or implicit. As we will see in Section 2.5, many recent gerrymandering cases concluded that existing jurisdictional boundaries were unduly split when determining whether a districting plan is gerrymandered. Importantly, county and city boundaries are non-partisan boundaries which are less likely to be split than more arbitrary boundaries at the census block level (like some generated by ensemble methods).

In the rare case of jurisdictional districts resultant from very high population counties, some very low population districts emerge from repeatedly splitting the county to lower its population (see Subsection 3.1 for more details on the application of this algorithm). The population of these districts then have a disproportionate (though largely insignificant) impact on the computed benchmark.

Additionally, the simplicity of Jurisdictional Partisan Advantage over ensemble measures means that it can sometimes be overly sensitive to close election outcomes. This is because jurisdiction populations are assigned to the majority party in a jurisdiction, regardless of the closeness of the race. In jurisdictions with very high populations, this sensitivity can affect overall measures of fairness of the evaluated plan (we see this in Section 3.3.3 and Section 3.3.11).

As a final important note, unlike the other measures, Jurisdictional Partisan Advantage uses jurisdiction populations to compute benchmark outcomes rather than vote totals. This means districts are assigned based on number of residents, as opposed to number of active voters.

2.5 Legal Context and Requirements for Redistricting

In this section, we consider four high profile court cases rejecting gerrymanders during the 2022 Redistricting cycle: the Ohio Supreme Court case invalidating a Republican gerrymander, *Adams v. DeWine*;⁵ the North Carolina Supreme Court case doing the same, *Harper v. Hall*;⁶ the Maryland

⁴*Harper v. Hall*, 868 S.E.2d 499, 511 (N.C.); Duchin Aff. at 9, February, 2022.

⁵*Adams v. DeWine*, Ohio St.3d, 2022-Ohio-89, N.E.3d (2022).

⁶*Harper v. Hall*, 868 S.E.2d 499, 511 (N.C. Feb. 14, 2022).

Anne Arundel County Circuit Court case invalidating a Democratic gerrymander, *Szeliga v. Lamone*,⁷ and the Supreme Court of New York Appellate Division case upholding that the New York districting plan was a Democratic gerrymander, *Harkenrider v. Hochul*.⁸ Of particular interest is the evidence of gerrymandering found persuasive by the judges and justices in these cases as that provides insight into the necessary traits of a legally effective measure of gerrymandering.

In sum, the maintenance of jurisdictional boundaries is of extreme importance in establishing a gerrymander in all but the New York case. Proportionality measures of fairness, such as the efficiency gap, are rejected by the dissenting justices in *Adams v. DeWine* and *Harper v. Hall*. Ensemble-based measures, while not outright rejected in any of the cases and used in all but *Harper v. Hall*, are seemingly misunderstood by the dissent in *Adams v. DeWine*, validating concerns about their complexity in court. In *Harkenrider v. Hochul*, the incapacity of ensemble measures to take into account all factors required by the New York Constitution is flagged, suggesting another limitation of such measures. In *Szeliga v. Lamone*, the importance of apolitical mathematical measures is stressed. Altogether, these cases demonstrate the value of a measure like Jurisdictional Partisan advantage—it is at its best when existing political jurisdictions are maintained, it does not presuppose proportionality as a proxy for districting fairness, it yields an apolitical mathematical benchmark, and it is simpler to convey and justify than ensemble-based measures.

2.5.1 *Adams v. DeWine*

In *Adams v. DeWine*, a group of Ohio voters sued the governor and other elected officials, asserting that the enacted congressional districting plan violated the Ohio Constitution. They challenged the plan under Article XIX, §1(C)(3)(a) and §1(C)(3)(b). §1(C)(3)(a) states:

The general assembly shall not pass a plan that unduly favors or disfavors a political party or its incumbents.

Additionally, §1(C)(3)(b) states:

The general assembly shall not unduly split governmental units, giving preference to keeping whole, in the order named, counties, then townships and municipal corporations.

Article XIX was added to the Ohio Constitution in 2018 via a ballot initiative titled “Congressional Districting Procedures Amendment,” and was designed to reduce gerrymandering in the state. A 4-3 majority of the court, the Democrats joined with Chief Justice Maureen O’Connor, found that the districting plan was indeed in violation of both §1(C)(3)(a) and §1(C)(3)(b) of Article XIX. The primary evidence described in the opinion relies on ensemble-based measures. Dr. Kosuke Imai of Harvard University generated 5,000 Article XIX compliant plans, finding that the number of seats won by Republicans was less in all ensemble plans than in the enacted plan.⁹ Dr. Jowei Chen of the University of Michigan simulated another 1,000 Article XIX compliant plans and found the enacted plan to be a statistical outlier.¹⁰ The courts concluded that this evidence demonstrated “beyond a reasonable doubt that the enacted plan excessively and unwarrantedly favor[ed] the Republican party and disfavor[ed] the Democratic party.”¹¹ The petitioners’ expert witnesses also demonstrated, via ensemble methods, that several Ohio urban counties were unduly split to disproportionately favor the Republican party.¹²

The majority also accepted evidence noting that the efficiency gap, partisan symmetry, and two related measures of this plan are statistical outliers when compared to historical election results across the country.¹³

In this case, the majority favorably received ensemble-based measures. They also accepted partisan symmetry and efficiency gap related measures when the measures demonstrate that the map is a statistical outlier. However, not all states have majorities willing to accept political science measures of gerrymandering nor do all states have provisions in their constitutions prohibiting gerrymandering. It is hence crucial to understand what elements of the expert testimony were not compelling to the dissenting justices.

⁷*Szeliga v. Lamone*, C-02-CV-21-001816 (Md. Cir. Ct., 2022).

⁸*Harkenrider v Hochul*, Sup. Ct. Steuben County Index No. E2022-0116cv Doc# 243 (2022).

⁹*Adams v. DeWine*, Ohio St.3d, 2022-Ohio-89, N.E.3d, ¶49.

¹⁰*Id* at ¶50.

¹¹*Id* at ¶51.

¹²*Id* at ¶62.

¹³*Id* at ¶63, 64, 65.

The dissenting justices in this case had a less generous interpretation of the ambiguous word “unduly” in §1(C)(3)(a) and §1(C)(3)(b) of Article XIX. They note that “by prohibiting only ‘undue’ favoritism, §1(3)(a) presupposes that *some* degree of partisan favoritism and *some* amount of governmental-unit splitting is acceptable” [emphasis in the original].¹⁴

They go on to question whether there is truly an obvious benchmark for measuring “unduly.” They discount the efficiency gap as a reasonable benchmark by noting that if every precinct in Ohio had the same political demographics as the state as a whole, Republicans would win every precinct, regardless of the map drawn, but the maps would still be flagged in terms of proportionality. They claim that the majority is implicitly concluding that the plan unduly favors Republicans “as compared to the results that would be obtained if we followed a system of proportional representation.”¹⁵ This implicit addition, they argue, is not based in the text of the Ohio Constitution.

Critically, it appears that complexity played an adverse role here. The core idea of ensemble analyses, that a fair plan should not differ significantly in outcomes from an average legal plan, does not appear to have been effectively conveyed to the justices. In describing the statistical measures introduced by the petitioners’ experts, the dissenting justices state that “they all use as their baseline the idea that a plan is fair when it achieves a result that resembles proportional representation.” However, the baseline established by ensemble-based measures is instead the outcome of a *random legal map*.¹⁶ This is problematic as, although ensemble-based measures are likely the most mathematically rigorous currently-existing methods of quantifying gerrymandering, they are by far the most nuanced. It appears their nuances were not effectively conveyed to the dissenting justices here, demonstrating that concerns about the mathematical complexity of gerrymandering measures are well-founded.

2.5.2 *Harper v. Hall*

In *Harper v. Hall*, North Carolina voters sued state representatives, senators, and election officials, asserting that the districting plans drawn by those officials were unlawful partisan gerrymanders in violation of the North Carolina Constitution. Unlike Ohio, the North Carolina Constitution does not have an article specifically outlawing gerrymandering. Nonetheless, in this case a 4-3 majority of the court, the four Democratic justices vs. the three Republican justices, found that the North Carolina maps were illegal. In particular, they were found to be in violation of the free elections, equal protection, free speech, and freedom of assembly clauses of the North Carolina Constitution. In the opinion, the majority lists “multiple reliable ways of demonstrating the existence of an unconstitutional gerrymander.”¹⁷ These include: efficiency gap analysis, partisan symmetry analysis, and other related measures. Moreover, the majority supplies a method to analyze these measures.

If some combination of these metrics demonstrates there is a significant likelihood that the districting plan will give voters of all political parties substantially equal opportunity to translate votes into seats across the plan, then the plan is presumptively constitutional.¹⁸

The majority also stresses the importance of maintaining existing jurisdictional boundaries in complying with the court order, requiring the maintaining of whole counties (which is required by the North Carolina Constitution, for example, Article II, §5(3)).

As before, of interest is the opinion of the three dissenting Republican justices. Unlike in *Adams v. DeWine*, the dissenting justices in this case do not await a better measure before concluding that such partisan gerrymandering cases are nonjusticiable as in *Rucho v. Common Cause*.¹⁹ However, in essence, their complaint with the majority is the same as in *Adams v. DeWine*. They reject an implicit “statewide proportionality requirement” (in essence, a low efficiency gap score).²⁰ They go further to echo the views of the U.S. Supreme Court in *Rucho v. Common Cause*, concluding that political science models are unreliable since they cannot encapsulate the complicated reality of all individual voters.

In many ways, the opinion of the dissent in this case is alarming. Election outcomes result from aggregate behavior and can be predicted with little attention to individual preference. Nonetheless,

¹⁴*Id* at ¶137.

¹⁵*Adams v. DeWine*, Ohio St.3d, 2022-Ohio-89, N.E.3d, ¶143.

¹⁶*Id* at ¶165.

¹⁷*Harper v. Hall*, 868 S.E.2d 499, 511 (N.C. Feb. 14, 2022), 6.

¹⁸*Id* at pg. 7.

¹⁹*Rucho v. Common Cause*, 139 S. Ct. at 2503-04.

²⁰*Harper v. Hall*, 868 S.E.2d 499, 511 (N.C. Feb. 14, 2022), 12.

without a change of heart, justices sharing this perspective seem adamant to let gerrymandering abound in the absence of laws passed by a congress elected on gerrymandered maps. This is a disheartening reminder of the urgency of establishing more durable solutions to gerrymandering. As we will see in Section 3.4, having non-politicians draw the maps appears to be a step in the right direction.

2.5.3 *Szeliga v. Lamone*

In *Szeliga v. Lamone*, a group of Republican voters and Republican legislators sued the Maryland Board of Elections, attesting that the enacted congressional plan was a gerrymander in violation of the Maryland Constitution and Declaration of Rights.

We consider here the evidence found compelling in this case. Expert witness Sean Trende counted the number of county splits in historical maps and found that the 2021 map had 17 county splits. He compared this to the 21 in 2002 and 2012 and 8, 10, and 13 in 1972, 1982, and 1992, respectively. The number of county splits was then deemed historically high by the court. Whether or not that is an effective analysis of the available data, the fact that county splits were important is demonstrative of the central role of maintaining existing jurisdictions in establishing gerrymanders in the courts. Mr. Trende also provided compactness scores of the enacted map, as compared to historical maps, and asserted that the map was an outlier, although the details of that analysis are not provided in the memorandum.²¹ Additionally, Mr. Trende provided an ensemble analysis of the map, finding that, using 2020 presidential election data, only 4.4% of the maps in the ensemble yielded the 8 Democrat seat outcome of the enacted map. While other experts testified in this case, the other expert testimonies were less impactful than Mr. Trende's, as his testimony was "undergirded with empirical data that could be reliably tested and validly replicated."²² In contrast to the dissent in *Harper v. Hall*, this demonstrates that some courts do favorably weigh mathematical quantification of redistricting fairness, in sharp contrast with the conclusions of *Rucho v. Common Cause*.

2.5.4 *Harkenrider v. Hochul*

In this case, a group of New York voters sued New York's Governor, lieutenant Governor, and various election officials, asserting that the original redistricting plan enacted by the state was a gerrymander in violation of the New York Constitution. A lower court agreed with this assertion, and this ruling was upheld by the Appellate Division of the New York Supreme Court.

In particular, the court found that the evidence of a one-party districting process, the differing outcomes between the 2012 and 2022 redistricting maps, and the analysis by Sean Trende (the same expert as in *Szeliga v. Lamone*) were compelling. In particular, Mr. Trende again used ensemble analyses to claim that the outcomes with this map were an outlier in the space of all fair maps.²³

The dissent in this case rejected that the petitioners offered sufficient proof of a gerrymander, given that the methods employed by Mr. Trende relied on a not yet peer-reviewed paper.²⁴ This appears a legitimate concern given the similar status of many ensemble methods as of this writing. Additionally, the ensembles did not account for all the criteria required by the New York Constitution: including, to name a few requirements, maintaining communities of interest, protecting racial and minority voting rights, and adhering to the Federal Voting Rights Act. This implies that claiming an ensemble of maps truly includes only legal maps may require some amount of validation in courts across the country, a distracting, detracting, and complicating factor for such methods.

3 Results

3.1 Methodology

In this section we describe precisely how the results displayed Table 1 were computed. All simulated election results were computed using Dave's Redistricting App (DRA). We use DRA for the combination of accessibility and ease of computation.

²¹ *Szeliga v. Lamone*, C-02-CV-21-001816 (Md. Cir. Ct., 2022), 61.

²² *Id* at pg. 83

²³ *Harkenrider v Hochul*, Sup. Ct. Steuben County Index No. E2022-0116cv Doc# 243 (2022), 5.

²⁴ *Id* at pg. 9.

State	Fair Benchmark		Map Seats		Part. Adv.	Rel. Adv.	# Seats	# Elections
	DEM	REP	DEM	REP	(REP)	(REP)		
AL	1.90	5.10	1.00	6.00	0.90	0.06	7	6
AK			N/A				1	N/A
AZ	3.71	5.29	3.33	5.67	0.38	0.00	9	6
AR	0.75	3.25	0.00	4.00	0.75	0.06	4	5
CA	42.92	9.08	44.33	7.67	-1.41	-0.02	52	3
CO	5.09	2.91	5.00	3.00	0.09	0.00	8	5
CT	4.61	0.39	4.60	0.40	0.01	0.00	5	5
DE			N/A				1	N/A
FL	14.51	13.49	9.40	18.60	5.11	0.16	28	5
GA	7.06	6.94	5.00	9.00	2.06	0.11	14	6
HI	2.00	0.00	2.00	0.00	0.00	0.00	2	5
ID	0.18	1.82	0.00	2.00	0.18	0.00	2	5
IL	12.08	4.92	14.00	3.00	-1.92	-0.08	17	5
IN	2.90	6.10	2.00	7.00	0.90	0.04	9	6
IA	1.40	2.60	0.60	3.40	0.80	0.07	4	5
KS	1.27	2.73			N/A		4	5
KY	1.92	4.08	1.67	4.33	0.26	0.00	6	3
LA	1.72	4.28	1.00	5.00	0.72	0.04	6	5
ME*	1.02	0.98	1.00	1.00	0.02	0.00	2	4
MD	5.34	2.66	6.20	1.80	-0.86	-0.04	8	5
MA	7.09	1.91	7.40	1.60	-0.31	0.00	9	5
MI	7.28	5.72	7.20	5.80	0.08	0.00	13	5
MN	4.94	3.06	4.83	3.17	0.11	0.00	8	6
MS	1.13	2.87	1.00	3.00	0.13	0.00	4	7
MO	3.00	5.00			N/A		8	6
MT	0.80	1.20	0.33	1.67	0.46	0.00	2	6
NE	1.11	1.89	0.40	2.60	0.71	0.07	3	5
NV	3.01	0.99	3.00	1.00	0.01	0.00	4	5
NH	0.90	1.10			N/A		2	7
NJ	9.02	2.98	9.60	2.40	-0.58	-0.01	12	5
NM	2.12	0.88	3.00	0.00	-0.88	-0.13	3	5
NY	21.56	4.44			N/A		26	5
NC	7.01	6.99	6.67	7.33	0.35	0.00	14	6
ND			N/A				1	N/A
OH	6.29	8.71	4.80	10.20	1.49	0.07	15	5
OK	0.30	4.70	0.00	5.00	0.30	0.00	5	5
OR	3.77	2.23	4.17	1.83	-0.40	0.00	6	6
PA	9.22	7.78	8.80	8.20	0.42	0.00	17	5
RI	1.88	0.12	2.00	0.00	-0.12	0.00	2	5
SC	1.59	5.41	1.00	6.00	0.59	0.01	7	5
SD			N/A				1	N/A
TN	2.17	6.83	1.10	7.90	1.07	0.06	9	5
TX	17.66	20.34	13.60	24.40	4.06	0.09	38	5
UT	0.54	3.46	0.00	4.00	0.54	0.01	4	6
VT			N/A				1	N/A
VA	7.23	3.77	6.80	4.20	0.43	0.00	11	5
WA	7.26	2.74	6.67	3.33	0.59	0.01	10	6
WV*	0.00	2.00	0.00	2.00	0.00	0.00	2	1
WI	3.43	4.57	2.60	5.40	0.83	0.04	8	5
WY			N/A				1	N/A

Table 1: State Jurisdictional Partisan Advantage

To compute enacted map seats, we first upload a shapefile, geoJSON, or census block equivalency file to DRA. Then, we use the “Show District Statistics” feature with all DRA-available presidential, gubernatorial, and US Senate election results from 2016–2021. In the case of elections with runoff results, we use only the runoff results. In the special case of West Virginia, we use the only DRA-available election, “PRES 2012/2016.” Additionally, the 2018 Maine US Senate election is available on DRA, but we ignore this election as independent Angus King won, complicating the two-party aggregate analyses for a single race. In every simulated election, we select “Total Population 2020” under the census data option. The number of elections used from DRA is listed in the “# Elections” column of Table 1.

For a given set of simulated election results, we tally the number of districts in which each party won a majority percentage of the Democratic-Republican vote. The average of these tallies over the elections used are the results in the “Map Seats” column of Table 1. The total number of seats allocated to the state is listed in the “# Seats” column of Table 1.

The computation of the “Fair Benchmark” column is somewhat more involved. For each state, we construct the jurisdictional map as follows. We begin with the map of state counties. Then, we set a threshold of twice the average population of a state congressional district for the jurisdictional districts. The thresholds are determined by adjusted populations from Summer 2021 to be as in line with current data as possible while still preceding all states’ redistricting processes. Any jurisdictional district with population greater than twice the state population divided by the number of congressional districts should be sub-divided. For each jurisdictional district exceeding the population threshold, the portion of cities geographically contained in the county are repeatedly made into their own jurisdictional districts (which will not be further subdivided) until either there are no more cities or the remaining population is below the threshold.

For example, Clark County, Nevada had a population of 2,265,461 according to the 2020 census, above the 1,554,231 threshold for Nevada. The most populous city contained in the county is Las Vegas, so Las Vegas is made into its own jurisdiction. The remaining population of Clark County, 1,623,558, is still above the threshold, so the next most populous city in the county, Henderson, is also made into its own jurisdiction. Then the remaining population of Clark County is 1,305,948, less than the threshold, and the remaining portion of the county becomes another jurisdiction. The resulting three jurisdictions are shown in Figure 2.

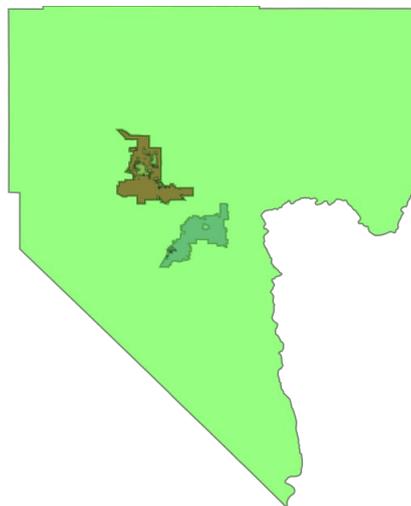


Figure 2: Clark County split

Note that, although in many states cities are entirely contained in single counties, this is not always the case, such as in Texas. In that case subdivision requires working at the census block level, a convenient feature of DRA. Moreover, in some cases, the “cities” listed by DRA differ from the cities we use for subdivision. For example, some regions listed as cities by DRA are instead townships.

Once the complete jurisdictional map is algorithmically generated, we simulate elections on it using DRA just as we do for the enacted maps. Then, we assign the population of each district to the party that wins the majority of the Democratic-Republican vote for each given election. The summed assigned population divided by the total population yields the proportion of seats assigned to each party in each benchmark computation. The results shown in the “Fair Benchmark” columns

of Table 1 are the averages of the seats assigned to each party in the benchmark computations over all elections used.

The “Part. Adv.” column is the difference between the “Map Seats” and “Fair Benchmark” column for the Republican party. In each election simulation, the relative advantage is a simple manipulation of the partisan advantage. If the partisan advantage has a magnitude less than 0.5, then it is considered trivial and yields a relative advantage of 0. Otherwise, the relative advantage is the partisan advantage decreased in magnitude by 0.5 and divided by the number of total seats in the state. This manipulation is to make the results in this column more indicative of actual advantage. In essence, it amounts to allowing 0.5 seats of rounding error before declaring a plan unfair. The results displayed in the “Rel. Adv.” columns of Table 1 are the average relative advantage computed over all DRA elections used for a given state.

Note that, in actually computing these results, complete jurisdictional maps were not computed but rather pieced together. This was merely a computational shortcut and has no effect on the results. For convenience, we provide a complete list of the 20 counties which exceed their state thresholds and which were split in their states’ final jurisdictional maps (note that many of the boroughs of New York City exceed the New York state threshold but cannot be split since they do not strictly contain any cities):

- | | |
|--------------------------------------|-------------------------------------|
| 1. Maricopa County, Arizona | 11. Miami-Dade County, Florida |
| 2. Alameda County, California | 12. Cook County, Illinois |
| 3. Los Angeles County, California | 13. Middlesex County, Massachusetts |
| 4. Orange County, California | 14. Wayne County, Michigan |
| 5. Riverside County, California | 15. Clark County, Nevada |
| 6. Sacramento County, California | 16. Bexar County, Texas |
| 7. San Bernardino County, California | 17. Dallas County, Texas |
| 8. San Diego County, California | 18. Harris County, Texas |
| 9. Santa Clara County, California | 19. Tarrant County, Texas |
| 10. Broward County, Florida | 20. King County, Washington |

3.2 Jurisdictional Partisan Advantage

In sum, we find that, across the country, Republicans have a Jurisdictional Partisan Advantage (PAJ) of **17.88 seats**. Note that this statistic assumes the partisan advantage contributed by the unfinished districting processes in Missouri, New Hampshire, and New York is zero.

Out of the 41/44 states to enact new districting plans, the plans of 20 states yield a Jurisdictional Partisan Advantage of at least a half. Of these, 15 yield an advantage for Republicans and 5 for Democrats. By number of seats, the seven states with more than one seat of Jurisdictional Partisan Advantage are the following:

- | | |
|----------------------|------------------------|
| 1. Florida (+5.11) | 5. Ohio (+1.49), |
| 2. Texas (+4.06), | 6. California (-1.41), |
| 3. Georgia (+2.06), | 7. Tennessee (+1.07). |
| 4. Illinois (-1.92), | |

While this statistic is important for the aggregate total advantage, it implies that large states with a small advantage are as unfair as much smaller states with a similar advantage (California with its 52 seats versus Tennessee with its 9, for example). The seven states with at least 5 congressional seats with the most relative partisan advantage (as shown in the table) are instead:

- | | |
|----------------------|-----------------------|
| 1. Florida (+16.5%) | 5. Ohio (+6.6%), |
| 2. Georgia (+11.1%), | 6. Tennessee (+6.3%), |
| 3. Texas (+9.4%), | 7. Alabama (+5.7 %) |
| 4. Illinois (-8.4%), | |

We call this the relative partisan advantage as it is comparable across the states. In particular, California has a mere -1.7% relative partisan advantage. Thus, barring New Mexico with a -12.6% relative partisan advantage likely resultant from only having 3 total seats, in some sense, Illinois and Maryland are the only states with districting plans significantly tilted in favor of Democrats in this cycle, at least according to the Jurisdictional Partisan Advantage measure. Meanwhile, there are nine states with a more than 2% relative partisan advantage and at least 5 congressional districts: those above as well as Indiana (+4.5%), Wisconsin (+4.2%), and Louisiana (+3.6%).

As interesting as those included above are those that are missing. Jurisdictional partisan advantage is referred to as artificial partisan advantage in Eguia 2021 and Table A.9 in the appendix includes a list of average computed Jurisdictional Partisan Advantage for the initial maps from last redistricting cycle (apart from Florida and Maine). Among the states in that table, the ones with at least one seat of partisan advantage are:

- | | |
|---------------------------|-----------------------|
| 1. North Carolina (+2.54) | 7. California (-1.52) |
| 2. Texas (+2.22) | 8. Illinois (-1.50) |
| 3. Ohio (+2.14) | 9. New York (+1.33) |
| 4. Pennsylvania (+1.85) | 10. Georgia (+1.25) |
| 5. Michigan (+1.58) | 11. Maryland (-1.19) |
| 6. Virginia (+1.57) | 12. Utah (+1.15) |

There were 12 states with at least one seat of partisan advantage as compared to the seven for this cycle. Among those 12, only California, Illinois, and Maryland yielded an advantage for Democrats. In total, Eguia 2019 computed an advantage of 16.55 seats for Republicans.

Additionally, none of North Carolina, Michigan, Pennsylvania, Virginia, or Utah have more than one seat worth of partisan advantage in this cycle and Tennessee has more than one seat of partisan advantage this cycle, but had minimal advantage last cycle.

Of note, these changes did not occur naturally. Initial North Carolina plans provided +3.01 seats of partisan advantage for Republicans, but the plan was thrown out by the North Carolina Supreme Court.²⁵ Michigan's maps were drawn by the Michigan Citizens Independent Redistricting Commission, a nonpartisan commission created by a ballot initiative in 2018. The congressional maps drawn by the commission this cycle yield almost no partisan advantage. Pennsylvania's maps were drawn by the state Supreme Court after the Republican-controlled legislature was unable to pass Republican-biased maps through the Democratic governor's veto. In Virginia, the state Supreme Court ended up drawing the maps after the bipartisan redistricting commission failed to come to an agreement by their deadline. Utah maps are still Republican-biased, though to a lesser extent. We analyze the outcomes of maps drawn by different authors in more detail in Section 3.4.

Moreover, although Republicans still have a significant advantage in Ohio, the enacted maps are a result of the initial maps being invalidated by the Ohio Supreme Court²⁶. Those preliminary maps offered Republicans an advantage of 2.09 seats.

A similar outcome arose in Maryland, where the original Democrat gerrymander was rejected by the courts, only to be replaced with a map yielding a milder Democrat advantage. The original maps provided 1.26 seats of advantage for Democrats.

Overall, it appears that most of the aggregate unfairness in this redistricting cycle is concentrated in fewer states, like Florida, Texas, Georgia, and Illinois. The outcome still is substantially in favor of Republicans with most significantly biased maps favoring Republicans.

²⁵*Harper v. Hall*, 868 S.E.2d 499, 511 (N.C. Feb. 14, 2022).

²⁶*Adams v. DeWine*, Ohio St.3d, 2022-Ohio-89, N.E.3d.

3.3 Comparison to Other Measures

We also computed the partisan advantage in each state using several other commonly used metrics, the three proportionality measures described in Section 2.2. As a reminder, $PA|J$, $PA|DB$, $PA|^2$, $PA|^3$ are partisan advantage measured against the jurisdictional, double bonus, votes to seats squared, and votes to seats cubed measures, respectively.

The aggregate results are summarized in Table 2. For addition results, see Table 3 in the Appendix.

	$PA J$	$PA DB$	$PA ^2$	$PA ^3$
Agg. Rep. Adv.	17.88	9.40	6.85	12.26

Table 2: Comparison of Aggregate Partisan Advantage Statistics

All four measures indicate a substantial aggregate advantage for Republicans. Interestingly, the aggregate Jurisdictional Partisan Advantage suggests the highest partisan advantage of the four measures. Indeed, although the Jurisdictional Partisan Advantage discounts the natural Republican geographical advantages, the Jurisdictional Partisan Advantage is sensitive to counties won by razor thin margins unlike the other three measures we compare to here (but all four are sensitive to seats won by slim margins). As a result, Jurisdictional Partisan Advantage often overestimates the advantage yielded by extremely gerrymandered maps where only one party wins all of the close elections. This can be fixed with sensitivity analysis.

Several states have considerable variance between the measures. We analyze these disparities in detail on a case by case basis.

3.3.1 California

By each of the $PA|J$, $PA|DB$, and $PA|^2$ measures, California has a considerable absolute seat advantage for Democrats. The advantage suggested by the $PA|J$ is small, much smaller than that implied by the $PA|DB$ and $PA|^2$ measures. Indeed, by the $PA|^2$ measure, California is the most unfair state in the nation. However, a known flaw of the efficiency gap (and by their close similarity, $PA|^2$ and $PA|DB$) is that it over-estimates partisan advantage in particularly partisan states. The prototypical example here is that the efficiency gap will imply that almost any map in Massachusetts is a Democratic gerrymander, simply by virtue of the population distribution of Republicans in the state. California, by percent Democratic vote, has the second highest Democratic vote percentage of states considered in this paper, with only Hawaii having a higher vote percentage.

It is challenging to draw substantial conclusions about the outcomes in California since DRA only included three elections worth of data. Nonetheless, it is interesting to note that the jurisdictional advantage measure suggests that Republicans are actually advantaged in the presidential elections. One possible explanation is that Republicans win many jurisdictions by very thin margins with the 2018 gubernatorial election data. Indeed, this is the case. Increasing Democratic vote percentage by 3% and decreasing Republican vote percentage by the same amount does not flip a single district in the enacted plan. However, it yields a jurisdictional benchmark of 45.1 seats for Democrats, yielding a Jurisdictional Partisan Advantage of +1.1, similar to in the other two elections. This suggests that, if anything, California is likely slightly biased Republican, not a Democratic gerrymander, at least according to Jurisdictional Partisan Advantage.

3.3.2 Connecticut

Connecticut has no partisan advantage according to the jurisdictional benchmark, but is a Democratic gerrymander according to the others. For context, Connecticut is a heavily Democratic Northeastern state which sometimes votes Republican for gubernatorial races. In all but the 2018 gubernatorial race, the only counties with Republicans winning a majority of the vote were Litchfield and Windham counties, two of the somewhat smaller counties in the state. In four of the five races analyzed (all but the 2016 Senate election which was a Democratic sweep), each party won between 39–63% of the vote in every county. Thus, Republicans are somewhat evenly distributed across the state. Given that there is a majority of Democratic voters overall, this makes it much more difficult for Republicans to win a number of seats proportional to their vote percentage. In some sense, proportionality is the outcome assumed as fair by $PA|DB$ and, to a lesser extent, $PA|^2$ and $PA|^3$. This geo-

graphic advantage is exactly what Jurisdictional Partisan Advantage is designed to exclude, which it appears to do great effect in this instance, explaining the disparity.

3.3.3 Florida

All four measures suggest that Florida is the worst gerrymander in the country. In terms of the relative partisan advantage, it is by far the most unfair.

Notably, PAJ implies more advantage than the other three measures, especially when considering the 2020 presidential, 2018 US Senate, and 2018 gubernatorial data. We analyze why this occurs.

It turns out that PAJ appears to be overestimating the partisan advantage in these cases due to Democrats narrowly winning highly populous counties without those county majorities translating into any district wins. This is caused by PAJ's sensitivity to close elections in which only one party wins all of the tightly contested elections.

In particular, adjusting the 2020 presidential election data such that Republicans win 2% more of the vote and Democrats 2% less, no districts flip in the enacted plan, but nearly 4,000,000 more residents are assigned to the Republican party when computing the jurisdictional advantage. These residents come almost exclusively from Duval, Pinellas, and Seminole counties, as well as the jurisdiction encompassing the area in Miami-Dade county excluding the most populous cities. With these adjusted election results, the PAJ of the map comes out to be 2.66 seats for the Republican party, in line with the other three measures.

Adjusting the election data in the 2018 US Senate race to give Republicans 2 % more of the vote yields a similar outcome. No districts flip in the enacted map, but nearly 2,000,000 residents worth of population move from being assigned to Democrats to being assigned to Republicans, mostly from Seminole, St. Lucie, and Duval counties. This yields a new PAJ of 4.00 seats for the Republican party. This is almost exactly the same advantage implied by the other three measures.

Likewise, adjusting the election data in the 2018 gubernatorial election to give Republicans 2% more of the vote yields the same outcome. Almost 2,000,000 residents worth of population flip, mostly concentrated this time in Pinellas, St. Lucie, and Seminole counties. No districts in the enacted plan flip. So, the new PAJ is 4.04 seats for the Republican party, similar to the advantage implied by the other three measures.

The sensitivity of the jurisdictional measure here is suggestive of an extremely efficient cracking of the Democratic vote. Republicans win many districts by a slim majority and only lose by a significant margin. Hence, adjusting the vote in the Republican's favor does not change the map outcomes, even though it has a dramatic impact on the jurisdictional benchmark.

Importantly, since the vote percentages are adjusted by such a small amount without changing outcomes on the enacted plans, this adjustment has a minimal impact on the non-jurisdictional measures relative to the impact on the jurisdictional measure. Hence, the comparison here is valid.

3.3.4 Georgia

In terms of relative partisan advantage, Georgia's maps are the second most unfair of any enacted so far, yielding an enormous +11.1% advantage for Republicans. Notably, the Jurisdictional Partisan Advantage measure indicates somewhat more unfairness than the other measures. Looking more closely at which elections cause this, the 2016 presidential election is the most significant culprit.

In 2020, Cobb county had a population of 766,149 and, among the elections we consider, is often the largest county with close vote percentages. In particular, in the 2016 presidential election, Democrats won Cobb county 48.89% to 46.69%, meaning a 1.5% change in vote outcomes shifts the jurisdictional benchmark from distributing the seats 6.26 : 7.74 (DEM:REP) to 7.56 : 6.44, without flipping a single seat in the enacted plan. Such a shift would reduce the Republican partisan advantage to only 1.26 in this case, in line with the other measures, without dramatically affecting to other three measures. This is a result of a similar phenomenon as in Florida (see Section 3.3.3).

3.3.5 Illinois

Illinois is a heavily Democratic state and, as such, the efficiency gap and related measures suggested higher partisan advantage than the Jurisdictional Partisan Advantage measure. The reasoning for this is similar to California (see Section 3.3.1).

3.3.6 Massachusetts

The disparity between the measures in Massachusetts is for largely the same reasons as the disparity in Connecticut (see Section 3.3.2). Indeed, Massachusetts is the classic example of the failures of proportionality and the outcomes are even more extreme than in Connecticut—the landslide election of Charlie Baker biases the average Democratic vote percentage to provide the illusion that Massachusetts has fewer typical Democratic voters than it actually does. Of all the other four elections analyzed, Republicans win a majority of the vote in only a single county, Plymouth county, in only the 2018 Senate race, irrespective of their overall 34.5% of the vote. As a result, the proportionality related measures over-estimate the fair number of Republican seats.

3.3.7 Nevada

According to jurisdictional partisan fairness, Nevada is quite fair. According to the other three measures, Nevada yields a considerable advantage of almost a seat for Democrats. The advantage is especially significant considering Nevada has four total seats. However, it is important to point out that measures which attempt to project fair seat shares based on vote shares are increasingly inaccurate as the population of a state decreases. This is because major population centers (which are often more Democratic-leaning) cannot be effectively divided to yield a outcome proportional to the vote distribution. In particular, although Nevada has a balanced vote percentage between the two parties, Democrats compose a majority of the vote in both Las Vegas and the rest of Maricopa County, besides Henderson, in all five elections measured. By itself, this makes up more than half the population of the state of Nevada. Combined with the fact that Democrats have a slim majority of Washoe county in all but the 2016 US Senate election and that is the third most populous jurisdiction (with only Henderson having a comparable population of those that remain), maps yielding a proportional outcome for Republicans would have to split jurisdictions that should naturally be unified. This disparity is an example of the capacity of the jurisdictional measure to take into account political geography.

3.3.8 New Jersey

See Connecticut and Massachusetts. The measures other than Jurisdictional Partisan Advantage underestimate the effects of political geography in states with evenly distributed minority parties. There are simply too few Republican leaning areas for the party to win seats proportionate to their vote share (Monmouth, Ocean, and Morris counties' combined 1.8 million of 9.3 million people notwithstanding).

3.3.9 Oklahoma

The disparity between the jurisdictional measure and the other measures in Oklahoma is comparable to Nevada. Both states have few seats and have Democratic voting areas condensed into highly populated areas. However, Oklahoma is considerably more Republican-leaning than Nevada on the whole. Indeed, the only election in which Democrats won a single county in Oklahoma was the 2018 gubernatorial race. Moreover, the population of Oklahoma is spread more evenly among the counties than in Nevada. This distribution makes it nearly impossible for Democrats to win a seat-share proportional to their vote share, as is accounted for by the jurisdictional measure but not the others.

3.3.10 South Carolina

The political geography of South Carolina makes it nigh impossible for Democrats to win more than two seats, regardless of the maps. This is clear by viewing the percent of total population won by Democrats in the jurisdictional calculations, ranging between 15.61% in the 2016 US Senate election to 25.02% in the 2018 gubernatorial race (with the average being closer to the outcome in the gubernatorial race). This geographic constraint is taken into account in PAJ but not the other measures.

3.3.11 Texas

Jurisdictional partisan advantage suggests that, at least in terms of sheer number of seats of advantage, Texas maps yield the second most partisan advantage of any state in the nation. Importantly, PAJ suggests considerably more seats of advantage than PA|DB, PA|², and PA|³.

The explanation here is similar to the explanation of the disparity between the measures in Florida (see Section 3.3.3). Republicans win all of the narrowly won districts in the enacted plan, while some jurisdictions have a small majority of Democratic voters. In this case, much of this disparity arises in considering the 2020 presidential and 2018 US Senate election data.

Giving Republicans 2% more of the vote in the 2020 presidential election does not flip any districts in the enacted plan but flips almost 3,000,000 residents worth of population in jurisdictions initially won by Democrats. The resulting PAJ is then 3.21, in line with the other measures.

Doing the same with the 2018 US Senate election data again does not flip a single district in the enacted plan, but shifts about 3.5 million residents from Democratic majority jurisdictions to Republican majority. The resulting PAJ is then 2.47, similar to the other measures.

Again, since the vote percentage is shifted by such a small amount without flipping districts in the enacted plan, the changes in the the other three measures are far less significant than the change in the Jurisdictional Partisan Advantage, making the advantage resultant in the shifted election comparable to the original other three measures.

3.3.12 Wisconsin

The disparity between the measures in Wisconsin is likely the result of the jurisdictional benchmark better accounting for Wisconsin's unique political geography. One way to see this is to look at the 2020 presidential election data. In this election, Republicans won less than half of the vote. Nonetheless, the jurisdictional benchmark awards them 5 of the 8 total seats in the state. A similar outcome can be observed in the 2016 presidential election. The proportionality measures are awarding Democrats more seats than are feasible given the geographic distribution of voters in the state.

3.4 A Comparison of Map Authors

In this section we compare the partisan fairness outcomes by map author, deferring to the map author classifications of Dave's Redistricting. According to their classification and filling in gaps as necessary (as of April 24th, 2022), the states are partitioned into four categories as follows.

The maps in the following states were drawn by independent redistricting commissions:

- | | |
|---------------|---------------|
| 1. Arizona | 5. Michigan |
| 2. California | 6. Montana |
| 3. Colorado | 7. Washington |
| 4. Idaho | |

The average Republican advantage among these states was -0.06 ± 0.70 seats (indicating one standard deviation here), meaning that, on average, these plans have minimal partisan advantage, but there are some yielding advantage. Namely, California provides advantage to Democrats while Washington and Montana provide a slight Republican advantage. It thus appears these voter initiatives have potential. An important caveat to this is that many more states now have such commissions, but the commissions failed due to outside influences. Such states include New York and Ohio, for example.

The maps in the following states were drawn by legislatures with a split control:

- | | |
|--------------|---------------|
| 1. Hawaii | 4. Nebraska |
| 2. Maine | 5. New Jersey |
| 3. Minnesota | 6. Wisconsin |

The average Republican advantage among these states was 0.18 ± 0.52 seats. Thus, on average, these split control plans were reasonably fair. However, the New Jersey plan yields advantage for Democrats while the Wisconsin and Nebraska plans both are tilted in favor of Republicans. Given there are only six total plans in this category and half are somewhat unfair, split legislatures appear somewhat unreliable sources of fair maps.

The last potential source of fair maps are those drawn by the courts. The courts do not have the authority to be the original source of maps, however, so this is mostly a test of the fairness of their remedial maps. The maps in the following states were drawn by state courts:

- | | |
|-------------------|-----------------|
| 1. Connecticut | 3. Pennsylvania |
| 2. North Carolina | 4. Virginia |

The average Republican advantage among these states was 0.30 ± 0.20 . Although this is the highest average thus far, it is notable that none of these maps have half a seat or more of advantage for either party and are thus deemed fair. This makes the courts the most reliable source of fair maps, followed by independent commissions.

Now we consider the maps drawn by party-dominated legislatures. Although ideally these would be fair, there is no expectation of their fairness. We begin by studying maps drawn by Republican-dominated state legislatures. The maps in the following states were drawn in Republican-dominated legislatures:

- | | |
|--------------|--------------------|
| 1. Alabama | 9. Mississippi |
| 2. Arkansas | 10. Ohio |
| 3. Florida | 11. Oklahoma |
| 4. Georgia | 12. South Carolina |
| 5. Indiana | 13. Tennessee |
| 6. Iowa | 14. Texas |
| 7. Kentucky | 15. Utah |
| 8. Louisiana | 16. West Virginia |

The average Republican advantage amongst these maps was $1.23 \pm .42$. However, this statistic is misleading here as not a single map yielded any Democratic advantage. Instead, the median of 0.78 seats of advantage for Republicans and interquartile range (range between 25th and 75th percentiles) of 0.48 - 1.28 seats is perhaps more illuminating. This means that the median Republican-drawn map yields a significant advantage for Republicans on the aggregate scale.

Next we view the maps drawn by the Democrat-dominated legislatures. The maps in the following states were drawn in Democrat-dominated legislatures:

- | | |
|------------------|-----------------|
| 1. Illinois | 5. New Mexico |
| 2. Maryland | 6. Oregon |
| 3. Massachusetts | |
| 4. Nevada | 7. Rhode Island |

The average Republican advantage among these maps was -0.60 ± 0.71 . Again, this statistic is misleading—no state yields any significant advantage to Republicans in this category (Nevada yields an insignificant 0.01 seat advantage). Instead, the median seat advantage is -0.35, with an IQR of -0.74 to -0.17. This IQR is similar to the Republican one, but somewhat less extreme.

The data in this section clearly suggests that party-dominated legislatures usually lead to maps biased toward that party. The data also reveals that court-drawn maps and commission maps from this cycle are reasonably fair in terms of the jurisdictional measure. Maps drawn by split-legislatures lie somewhere in between. Note that outcomes in this cycle may not be representative due to the lack of data points.

4 Conclusion

Jurisdictional partisan advantage is a promising measure for analyzing the partisan fairness of redistricting plans. It is effective at accounting for political geography in ways that elude comparably simple measures. However, sometimes it overestimates unfairness, especially for particularly unfair maps. This complication can be corrected for via sensitivity analyses.

In this paper, we find that, according to Jurisdictional Partisan Advantage measure, Republicans have a 17.88 seat advantage in the US House of Representatives from the drawing of redistricting maps. This is somewhat higher than comparable proportionality measures.

We also find that the courts and independent redistricting commissions are far more reliable sources of redistricting maps with minimal partisan advantage than legislatures dominated by a single party.

Appendices

A Comparison of Measures Table

In the below table, we show the computed measures for each Dave's Redistricting App election in each state. The "# Seats" column refers to the number of congressional districts in the state. The "Vote %" column displays GOP percent of the Republican-Democratic vote in that election. The Democratic vote can be inferred by subtracting this percentage from 100%. The next four columns display the benchmark number of Republican seats in the state according to that election data. The "DB" column refers to the Double Bonus measure (or 2:1 ratio Efficiency Gap). The " $(V : S)^2$ " column refers to the benchmark used in computing PAI^2 , the squared ratio measure. Likewise, the " $(V : S)^3$ " column refers to the benchmark used in computing PAI^3 . The "Juris." column refers to the jurisdictional benchmark. See the end of Section 2.2 for a more complete description of these benchmarks. The Democratic benchmark can be inferred by subtracting the Republican benchmark from the "# Seats" column. The following two columns (under "Map Seats") display the number of seats for Democrats and Republicans, respectively, under the enacted plan. Finally, the last four columns display the partisan advantage in each election with respect to each benchmark (see the end of Section 2.2 for an explanation of the notation). The rows with just the state abbreviation display the averages of each column over the elections. The "USA Total" row displays the aggregate results over all of the states for each column.

Table 3: Table of Gerrymandering Measures for Comparison

	# Seats	Vote % GOP	DB GOP	$(V : S)^2$ GOP	$(V : S)^3$ GOP	Juris. GOP	Map Seats DEM	Map Seats GOP	PA DB	PA ²	PA ³	PA J
USA Total	435	x	197.39	199.82	193.26	188.21	224.41	210.59	9.40	6.85	12.26	17.88
AL	7	60.11%	4.92	4.73	5.31	5.10	1.00	6.00	1.08	1.27	0.69	0.90
AL PRES 2020	7	62.91%	5.31	5.07	5.81	5.45	1.00	6.00	0.69	0.93	0.19	0.55
AL PRES 2016	7	64.51%	5.53	5.24	6.00	5.45	1.00	6.00	0.47	0.76	0.00	0.55
AL US SEN 2020	7	60.20%	4.93	4.78	5.43	5.43	1.00	6.00	1.07	1.22	0.57	0.57
AL US SEN 2016	7	64.20%	5.49	5.21	5.97	5.45	1.00	6.00	0.51	0.79	0.03	0.55
AL GOV 2018	7	59.60%	4.84	4.72	5.34	5.37	1.00	6.00	1.16	1.29	0.66	0.63
AL US SEN 2017	7	49.24%	3.39	3.39	3.34	3.45	1.00	6.00	2.61	2.61	2.66	2.55
AK	1						0.00	1.00	0.00	0.00	0.00	0.00
AZ	9	52.22%	4.90	4.87	5.07	5.29	3.33	5.67	0.77	0.80	0.60	0.38
AZ PRES 2020	9	49.84%	4.47	4.47	4.46	4.68	5.00	4.00	-0.47	-0.47	-0.46	-0.68
AZ PRES 2016	9	52.16%	4.89	4.88	5.08	4.99	2.00	7.00	2.11	2.12	1.92	2.01
AZ US SEN 2020	9	48.80%	4.28	4.29	4.18	4.68	5.00	4.00	-0.28	-0.29	-0.18	-0.68
AZ US SEN 2018	9	48.77%	4.28	4.28	4.17	4.68	4.00	5.00	0.72	0.72	0.83	0.32
AZ US SEN 2016	9	56.51%	5.67	5.60	6.18	7.36	2.00	7.00	1.33	1.40	0.82	-0.36
AZ GOV 2018	9	57.26%	5.81	5.71	6.36	5.34	2.00	7.00	1.19	1.29	0.64	1.66
AR	4	64.87%	3.19	3.01	3.44	3.25	0.00	4.00	0.81	-0.18	0.56	0.75
AR PRES 2020	4	64.21%	3.14	2.98	3.41	3.22	0.00	4.00	0.86	1.02	0.59	0.78
AR PRES 2016	4	64.12%	3.13	2.97	3.40	3.22	0.00	4.00	0.87	1.03	0.60	0.78
AR US SEN 2020	4	66.50%	3.32	3.10	3.55	3.36	0.00	4.00	0.68	0.90	0.45	0.64
AR US SEN 2016	4	62.29%	2.98	2.86	3.27	3.19	0.00	4.00	1.02	1.14	0.73	0.81
AR GOV 2018	4	67.25%	3.38	3.14	3.59	3.24	0.00	4.00	0.62	0.86	0.41	0.76
CA	52	35.98%	11.42	13.49	7.92	9.08	44.33	7.67	-3.75	-5.82	-0.25	-1.41
CA PRES 2020	52	35.09%	10.49	12.81	7.10	4.67	45.00	7.00	-3.49	-5.81	-0.10	2.33
CA PRES 2016	52	34.75%	10.14	12.56	6.82	6.89	44.00	8.00	-2.14	-4.56	1.18	1.11
CA GOV 2018	52	38.10%	13.62	15.10	9.83	15.67	44.00	8.00	-5.62	-7.10	-1.83	-7.67
CO	8	45.86%	3.34	3.37	3.04	2.91	5.00	3.00	-0.34	-0.37	-0.04	0.09
CO PRES 2020	8	43.06%	2.89	2.97	2.42	2.83	5.00	3.00	0.11	0.03	0.58	0.17
CO PRES 2016	8	48.47%	3.76	3.76	3.63	3.15	5.00	3.00	-0.76	-0.76	-0.63	-0.15
CO US SEN 2020	8	45.24%	3.24	3.27	2.88	2.91	5.00	3.00	-0.24	-0.27	0.12	0.09
CO US SEN 2016	8	48.04%	3.69	3.69	3.53	2.88	5.00	3.00	-0.69	-0.69	-0.53	0.12
CO GOV 2018	8	44.49%	3.12	3.17	2.72	2.79	5.00	3.00	-0.12	-0.17	0.28	0.21
CT	5	41.34%	1.63	1.73	1.35	0.39	4.60	0.40	-1.23	-1.33	-0.95	0.01
CT PRES 2020	5	39.80%	1.48	1.58	1.12	0.42	5.00	0.00	-1.48	-1.58	-1.12	-0.42
CT PRES 2016	5	43.05%	1.81	1.85	1.51	0.42	5.00	0.00	-1.81	-1.85	-1.51	-0.42
CT US SEN 2018	5	39.84%	1.48	1.59	1.13	0.26	5.00	0.00	-1.48	-1.59	-1.13	-0.26
CT US SEN 2016	5	35.69%	1.07	1.27	0.73	0.00	5.00	0.00	-1.07	-1.27	-0.73	0.00
CT GOV 2018	5	48.33%	2.33	2.34	2.25	0.85	3.00	2.00	-0.33	-0.34	-0.25	1.15

DE	1						1.00	0.00	0.00	0.00	0.00	0.00
FL	28	51.33%	14.75	14.72	15.11	13.49	9.40	18.60	3.85	3.88	3.49	5.11
FL PRES 2020	28	51.69%	14.95	14.93	15.42	12.19	8.00	20.00	5.05	5.07	4.58	7.81
FL PRES 2016	28	50.67%	14.38	14.37	14.56	15.25	11.00	17.00	2.62	2.63	2.44	1.75
FL US SEN 2018	28	50.10%	14.06	14.06	14.08	11.56	10.00	18.00	3.94	3.94	3.92	6.44
FL US SEN 2016	28	54.00%	16.24	16.15	17.30	16.78	8.00	20.00	3.76	3.85	2.70	3.22
FL GOV 2018	28	50.20%	14.11	14.11	14.17	11.67	10.00	18.00	3.89	3.89	3.83	6.33
GA	14	51.55%	7.43	7.40	7.62	6.94	5.00	9.00	1.57	1.60	1.38	2.06
GA PRES 2020	14	49.88%	6.97	6.97	6.95	6.47	5.00	9.00	2.03	2.03	2.05	2.53
GA PRES 2016	14	52.94%	7.82	7.80	8.22	6.43	5.00	9.00	1.18	1.20	0.78	2.57
GA US SEN 2020	14	49.39%	6.83	6.83	6.74	6.47	5.00	9.00	2.17	2.17	2.26	2.53
GA US SPSEN 2020	14	48.96%	6.71	6.71	6.56	6.47	5.00	9.00	2.29	2.29	2.44	2.53
GA US SEN 2016	14	57.41%	9.08	8.92	9.94	9.33	5.00	9.00	-0.08	0.08	-0.94	-0.33
GA GOV 2018	14	50.71%	7.20	7.20	7.30	6.50	5.00	9.00	1.80	1.80	1.70	2.50
HI	2	30.89%	0.25	0.39	0.18	0.00	2.00	0.00	-0.25	-0.39	-0.18	0.00
HI PRES 2020	2	34.97%	0.40	0.49	0.27	0.00	2.00	0.00	-0.40	-0.49	-0.27	0.00
HI PRES 2016	2	32.58%	0.30	0.42	0.20	0.00	2.00	0.00	-0.30	-0.42	-0.20	0.00
HI US SEN 2018	2	28.80%	0.15	0.33	0.12	0.00	2.00	0.00	-0.15	-0.33	-0.12	0.00
HI US SEN 2016	2	23.17%	0.00	0.21	0.05	0.00	2.00	0.00	0.00	-0.21	-0.05	0.00
HI GOV 2018	2	34.96%	0.40	0.49	0.27	0.00	2.00	0.00	-0.40	-0.49	-0.27	0.00
ID	2	66.17%	1.65	1.54	1.75	1.82	0.00	2.00	0.35	0.46	0.25	0.18
ID PRES 2020	2	65.88%	1.64	1.53	1.76	1.92	0.00	2.00	0.36	0.47	0.24	0.08
ID PRES 2016	2	68.20%	1.73	1.60	1.82	1.93	0.00	2.00	0.27	0.40	0.18	0.07
ID US SEN 2020	2	65.34%	1.61	1.52	1.74	1.92	0.00	2.00	0.39	0.48	0.26	0.08
ID US SEN 2016	2	70.39%	1.82	1.65	1.86	1.97	0.00	2.00	0.18	0.35	0.14	0.03
ID GOV 2018	2	61.02%	1.44	1.39	1.59	1.38	0.00	2.00	0.56	0.61	0.41	0.62
IL	17	41.69%	5.67	5.91	4.55	4.92	14.00	3.00	-2.67	-2.91	-1.55	-1.92
IL PRES 2020	17	41.34%	5.56	5.81	4.41	4.69	14.00	3.00	-2.56	-2.81	-1.41	-1.69
IL PRES 2016	17	41.56%	5.63	5.87	4.50	5.14	14.00	3.00	-2.63	-2.87	-1.50	-2.14
IL US SEN 2020	17	41.47%	5.60	5.85	4.46	4.69	14.00	3.00	-2.60	-2.85	-1.46	-1.69
IL US SEN 2016	17	42.49%	5.95	6.14	4.89	5.28	14.00	3.00	-2.95	-3.14	-1.89	-2.28
IL GOV 2018	17	41.59%	5.64	5.88	4.51	4.81	14.00	3.00	-2.64	-2.88	-1.51	-1.81
ID	9	57.24%	5.80	5.68	6.27	6.10	2.00	7.00	1.20	1.32	0.73	0.90
IN PRES 2020	9	58.20%	5.98	5.85	6.57	6.25	2.00	7.00	1.02	1.15	0.43	0.75
IN PRES 2016	9	60.15%	6.33	6.14	6.97	6.50	2.00	7.00	0.67	0.86	0.03	0.50
IN US SEN 2018	9	53.07%	5.05	5.04	5.32	5.58	2.00	7.00	1.95	1.96	1.68	1.42
IN US SEN 2016	9	55.13%	5.42	5.38	5.85	5.95	2.00	7.00	1.58	1.62	1.15	1.05
IN GOV 2020	9	63.77%	6.98	6.64	7.61	6.86	2.00	7.00	0.02	0.36	-0.61	0.14
IN GOV 2016	9	53.10%	5.06	5.04	5.33	5.46	2.00	7.00	1.94	1.96	1.67	1.54
IA	4	55.38%	2.43	2.40	2.59	2.60	0.60	3.40	0.97	1.00	0.81	0.80
IA PRES 2020	4	54.18%	2.33	2.32	2.49	2.40	0.00	4.00	1.67	1.68	1.51	1.60

IA PRES 2016	4	55.11%	2.41	2.39	2.60	2.39	0.00	4.00	1.59	1.61	1.40	1.61
IA US SEN 2020	4	53.40%	2.27	2.26	2.40	2.32	0.00	4.00	1.73	1.74	1.60	1.68
IA US SEN 2016	4	62.77%	3.02	2.89	3.31	3.81	0.00	4.00	0.98	1.11	0.69	0.19
IA GOV 2018	4	51.43%	2.11	2.11	2.17	2.10	3.00	1.00	-1.11	-1.11	-1.17	-1.10
KS	4	57.60%	2.61	2.53	2.79	2.73	-	-	-	-	-	-
KS PRES 2020	4	57.49%	2.60	2.55	2.85	2.44	-	-	-	-	-	-
KS PRES 2016	4	61.24%	2.90	2.80	3.19	3.61	-	-	-	-	-	-
KS US SEN 2020	4	56.00%	2.48	2.45	2.69	2.44	-	-	-	-	-	-
KS US SEN 2016	4	66.03%	3.28	3.08	3.52	3.61	-	-	-	-	-	-
KS GOV 2018	4	47.25%	1.78	1.79	1.67	1.58	-	-	-	-	-	-
KY	6	57.58%	3.91	3.79	4.15	4.08	1.67	4.33	0.42	0.54	0.18	0.26
KY PRES 2016	6	65.65%	4.88	4.58	5.25	4.53	1	5	0.12	0.42	-0.25	0.47
KY US SEN 2016	6	57.30%	3.88	3.81	4.24	4.38	2	4	0.12	0.19	-0.24	-0.38
KY GOV 2019	6	49.80%	2.98	2.98	2.96	3.32	2	4	1.02	1.02	1.04	0.68
LA	6	58.48%	4.02	3.90	4.34	4.28	1.00	5.00	0.98	1.10	0.66	0.72
LA PRES 2020	6	59.46%	4.14	4.03	4.56	4.45	1.00	5.00	0.86	0.97	0.44	0.55
LA PRES 2016	6	60.21%	4.22	4.10	4.66	4.45	1.00	5.00	0.78	0.90	0.34	0.55
LA US SEN 2020	6	63.42%	4.61	4.39	5.03	4.52	1.00	5.00	0.39	0.61	-0.03	0.48
LA US SEN 2016	6	60.65%	4.28	4.14	4.71	4.76	1.00	5.00	0.72	0.86	0.29	0.24
LA GOV 2019	6	48.66%	2.84	2.84	2.76	3.24	1.00	5.00	2.16	2.16	2.24	1.76
ME	2	48.63%	0.95	0.95	0.92	0.98	1.00	1.00	0.05	0.05	0.08	0.02
ME PRES 2020	2	45.33%	0.81	0.82	0.73	0.76	1.00	1.00	0.19	0.18	0.27	0.24
ME PRES 2016	2	48.55%	0.94	0.94	0.91	0.94	1.00	1.00	0.06	0.06	0.09	0.06
ME US SEN 2020	2	54.60%	1.18	1.18	1.27	1.50	1.00	1.00	-0.18	-0.18	-0.27	-0.50
ME GOV 2018	2	46.05%	0.84	0.85	0.77	0.71	1.00	1.00	0.16	0.15	0.23	0.29
MD	8	39.05%	2.25	2.54	1.95	2.66	6.20	1.80	-0.45	-0.74	-0.15	-0.86
MD PRES 2020	8	32.97%	1.28	1.74	0.85	1.67	7.00	1.00	-0.28	-0.74	0.15	-0.67
MD PRES 2016	8	36.85%	1.90	2.17	1.33	2.10	7.00	1.00	-0.90	-1.17	-0.33	-1.10
MD US SEN 2018	8	31.83%	1.09	1.62	0.74	1.50	7.00	1.00	-0.09	-0.62	0.26	-0.50
MD US SEN 2016	8	37.60%	2.02	2.26	1.44	2.86	7.00	1.00	-1.02	-1.26	-0.44	-1.86
MD GOV 2018	8	56.02%	4.96	4.90	5.39	5.16	3.00	5.00	0.04	0.10	-0.39	-0.16
MA	9	41.20%	2.92	3.15	2.57	1.91	7.40	1.60	-1.32	-1.55	-0.97	-0.31
MA PRES 2020	9	32.88%	1.42	1.95	0.95	0.00	9.00	0.00	-1.42	-1.95	-0.95	0.00
MA PRES 2016	9	35.52%	1.89	2.27	1.29	0.00	9.00	0.00	-1.89	-2.27	-1.29	0.00
MA US SEN 2020	9	33.30%	1.49	2.00	1.00	0.00	9.00	0.00	-1.49	-2.00	-1.00	0.00
MA US SEN 2018	9	37.47%	2.25	2.53	1.59	0.68	9.00	0.00	-2.25	-2.53	-1.59	-0.68
MA GOV 2018	9	66.80%	7.52	7.02	8.02	8.85	1.00	8.00	0.48	0.98	-0.02	-0.85
MI	13	47.92%	5.96	5.98	5.70	5.72	7.20	5.80	-0.16	-0.18	0.10	0.08
MI PRES 2020	13	48.59%	6.13	6.14	5.95	5.90	7.00	6.00	-0.13	-0.14	0.05	0.10
MI PRES 2016	13	50.16%	6.54	6.54	6.56	7.02	6.00	7.00	0.46	0.46	0.44	-0.02
MI US SEN 2020	13	49.13%	6.27	6.28	6.16	6.86	7.00	6.00	-0.27	-0.28	-0.16	-0.86
MI US SEN 2018	13	46.69%	5.64	5.67	5.22	4.52	8.00	5.00	-0.64	-0.67	-0.22	0.48

MI GOV 2018	13	45.05%	5.21	5.28	4.62	4.28	8.00	5.00	-0.21	-0.28	0.38	0.72
MN	8	44.78%	3.16	3.23	2.84	3.06	4.83	3.17	0.00	-0.06	0.33	0.11
MN PRES 2020	8	46.36%	3.42	3.44	3.139	3.55	4.00	4.00	0.58	0.56	0.86	0.45
MN PRES 2016	8	49.19%	3.87	3.87	3.805	3.85	3.00	5.00	1.13	1.13	1.19	1.15
MN US SEN 2020	8	47.13%	3.54	3.55	3.317	3.45	4.00	4.00	0.46	0.45	0.68	0.55
MN US SEN 2018	8	37.51%	2.00	2.25	1.423	1.49	7.00	1.00	-1.00	-1.25	-0.42	-0.49
MN US SENSP 2018	8	44.44%	3.11	3.16	2.709	3.29	5.00	3.00	-0.11	-0.16	0.29	-0.29
MN GOV 2018	8	44.03%	3.04	3.10	2.619	2.72	6.00	2.00	-1.04	-1.10	-0.62	-0.72
MS	4	56.17%	2.49	2.46	2.70	2.87	1.00	3.00	0.51	0.54	0.30	0.13
MS PRES 2020	4	58.38%	2.67	2.61	2.94	2.94	1.00	3.00	0.33	0.39	0.06	0.06
MS PRES 2016	4	59.49%	2.76	2.69	3.04	3.00	1.00	3.00	0.24	0.31	-0.04	0.00
MS US SEN 2020	4	55.09%	2.41	2.39	2.59	2.79	1.00	3.00	0.59	0.61	0.41	0.21
MS US SEN 2018	4	53.63%	2.29	2.28	2.43	2.79	1.00	3.00	0.71	0.72	0.57	0.21
MS US SENSP 2018	4	57.63%	2.61	2.56	2.86	3.01	1.00	3.00	0.39	0.44	0.14	-0.01
MS GOV 2019	4	52.79%	2.22	2.22	2.33	2.65	1.00	3.00	0.78	0.78	0.67	0.35
MO	8	55.66%	4.92	4.84	5.28	4.92	-	-	-	-	-	-
MO PRES 2020	8	57.84%	5.13	5.16	5.77	5.13	-	-	-	-	-	-
MO PRES 2016	8	60.04%	4.66	5.45	6.18	4.66	-	-	-	-	-	-
MO US SEN 2018	8	52.99%	4.80	4.46	4.71	4.80	-	-	-	-	-	-
MO US SEN 2016	8	51.67%	4.66	4.26	4.40	4.66	-	-	-	-	-	-
MO GOV 2020	8	58.38%	5.13	5.23	5.87	5.13	-	-	-	-	-	-
MO GOV 2016	8	53.05%	5.13	4.47	4.72	5.13	-	-	-	-	-	-
MT	2	54.56%	1.18	1.17	1.25	1.20	0.33	1.67	0.48	0.50	0.42	0.47
MT PRES 2020	2	58.40%	1.34	1.31	1.47	1.42	0.00	2.00	0.66	0.69	0.53	0.58
MT PRES 2016	2	61.08%	1.44	1.39	1.59	1.27	0.00	2.00	0.56	0.61	0.41	0.73
MT US SEN 2020	2	55.00%	1.20	1.19	1.29	1.27	0.00	2.00	0.80	0.81	0.71	0.73
MT US SEN 2018	2	48.20%	0.93	0.93	0.89	0.99	1.00	1.00	0.07	0.07	0.11	0.01
MT GOV 2020	2	56.67%	1.27	1.25	1.38	1.27	0.00	2.00	0.73	0.75	0.62	0.73
MT GOV 2016	2	48.03%	0.92	0.92	0.88	0.99	1.00	1.00	0.08	0.08	0.12	0.01
NE	3	62.93%	2.28	2.16	2.45	1.89	0.40	2.60	0.32	0.44	0.15	0.71
NE PRES 2020	3	59.78%	2.09	2.03	2.30	1.61	1.00	2.00	-0.09	-0.03	-0.30	0.39
NE PRES 2016	3	63.94%	2.34	2.22	2.54	1.61	0.00	3.00	0.66	0.78	0.46	1.39
NE US SEN 2020	3	72.01%	2.82	2.53	2.83	3.00	0.00	3.00	0.18	0.47	0.17	0.00
NE US SEN 2018	3	59.92%	2.10	2.04	2.31	1.61	0.00	3.00	0.90	0.96	0.69	1.39
NE GOV 2018	3	59.00%	2.04	1.99	2.25	1.61	1.00	2.00	-0.04	0.01	-0.25	0.39
NV	4	48.28%	1.86	1.87	1.79	0.99	3.00	1.00	-0.86	-0.87	-0.79	0.01
NV PRES 2020	4	48.78%	1.90	1.90	1.85	0.86	3.00	1.00	-0.90	-0.90	-0.85	0.14
NV PRES 2016	4	48.72%	1.90	1.90	1.85	0.86	3.00	1.00	-0.90	-0.90	-0.85	0.14
NV US SEN 2018	4	47.39%	1.79	1.80	1.69	0.86	3.00	1.00	-0.79	-0.80	-0.69	0.14
NV US SEN 2016	4	48.69%	1.90	1.90	1.84	1.49	3.00	1.00	-0.90	-0.90	-0.84	-0.49
NV GOV 2018	4	47.84%	1.83	1.83	1.74	0.86	3.00	1.00	-0.83	-0.83	-0.74	0.14
NH	2	51.26%	1.05	1.04	1.05	1.10	-	-	-	-	-	-

NH PRES 2020	2	46.25%	0.85	0.86	0.78	0.14	-	-	-	-	-	-
NH PRES 2016	2	49.79%	0.99	0.99	0.99	1.34	-	-	-	-	-	-
NH US SEN 2020	2	41.97%	0.68	0.70	0.55	0.00	-	-	-	-	-	-
NH US SEN 2016	2	49.95%	1.00	1.00	1.00	1.28	-	-	-	-	-	-
NH US GOV 2020	2	66.13%	1.65	1.54	1.76	2.00	-	-	-	-	-	-
NH US GOV 2018	2	53.55%	1.14	1.14	1.21	1.57	-	-	-	-	-	-
NH US GOV 2016	2	51.20%	1.05	1.05	1.07	1.34	-	-	-	-	-	-
NJ	12	42.76%	4.26	4.39	3.54	2.98	9.60	2.40	-1.86	-1.99	-1.14	-0.58
NJ PRES 2020	12	41.93%	4.06	4.22	3.28	2.36	10.00	2.00	-2.06	-2.22	-1.28	-0.36
NJ PRES 2016	12	43.18%	4.36	4.48	3.66	3.40	9.00	3.00	-1.36	-1.48	-0.66	-0.40
NJ US SEN 2020	12	41.69%	4.01	4.17	3.21	2.36	10.00	2.00	-2.01	-2.17	-1.21	-0.36
NJ US SEN 2018	12	44.21%	4.61	4.69	3.99	3.76	9.00	3.00	-1.61	-1.69	-0.99	-0.76
NJ GOV 2017	12	42.80%	4.27	4.40	3.54	3.01	10.00	2.00	-2.27	-2.40	-1.54	-1.01
NM	3	43.10%	1.09	1.12	0.93	0.88	3.00	0.00	-1.09	-1.12	-0.93	-0.88
NM PRES 2020	3	44.48%	1.17	1.19	1.02	0.91	3.00	0.00	-1.17	-1.19	-1.02	-0.91
NM PRES 2016	3	45.30%	1.22	1.23	1.09	0.91	3.00	0.00	-1.22	-1.23	-1.09	-0.91
NM US SEN 2020	3	46.87%	1.31	1.32	1.22	0.91	3.00	0.00	-1.31	-1.32	-1.22	-0.91
NM US SEN 2018	3	36.05%	0.66	0.78	0.46	0.74	3.00	0.00	-0.66	-0.78	-0.46	-0.74
NM GOV 2018	3	42.80%	1.07	1.10	0.89	0.91	3.00	0.00	-1.07	-1.10	-0.89	-0.91
NY	26	35.20%	5.31	6.53	3.87	4.44	-	-	-	-	-	-
NY PRES 2020	26	38.27%	6.90	7.62	5.00	6.61	-	-	-	-	-	-
NY PRES 2016	26	38.94%	7.25	7.89	5.36	7.42	-	-	-	-	-	-
NY US SEN 2018	26	33.00%	4.16	5.66	2.78	2.30	-	-	-	-	-	-
NY US SEN 2016	26	28.02%	1.57	4.08	1.45	0.29	-	-	-	-	-	-
NY GOV 2018	26	37.79%	6.65	7.42	4.76	5.59	-	-	-	-	-	-
NC	14	49.71%	6.92	6.92	6.88	6.99	6.67	7.33	0.42	0.41	0.45	0.35
NC PRES 2020	14	50.68%	7.19	7.19	7.29	6.91	7.00	7.00	-0.19	-0.19	-0.29	0.09
NC PRES 2016	14	51.95%	7.55	7.54	7.82	7.29	6.00	8.00	0.45	0.46	0.18	0.71
NC US SEN 2020	14	50.94%	7.26	7.26	7.40	6.96	7.00	7.00	-0.26	-0.26	-0.40	0.04
NC US SEN 2016	14	46.99%	6.16	6.18	5.75	7.37	6.00	8.00	1.84	1.82	2.25	0.63
NC GOV 2020	14	47.72%	6.36	6.38	6.05	6.68	7.00	7.00	0.64	0.62	0.95	0.32
NC GOV 2016	14	49.95%	6.99	6.99	6.98	6.72	7.00	7.00	0.01	0.01	0.02	0.28
ND	1						0.00	1.00				
OH	15	53.66%	8.60	8.51	9.02	8.71	4.80	10.20	1.60	1.69	1.18	1.49
OH PRES 2020	15	54.08%	8.72	8.67	9.30	8.66	4.00	11.00	2.28	2.33	1.70	2.34
OH PRES 2016	15	54.50%	8.85	8.79	9.48	8.65	4.00	11.00	2.15	2.21	1.52	2.35
OH US SEN 2018	15	46.60%	6.48	6.51	5.99	6.77	9.00	6.00	-0.48	-0.51	0.01	-0.77
OH US SEN 2016	15	61.24%	10.87	10.49	11.97	11.08	2.00	13.00	2.13	2.51	1.03	1.92
OH GOV 2018	15	51.91%	8.07	8.06	8.35	8.39	5.00	10.00	1.93	1.94	1.65	1.61
OK	5	66.32%	4.13	3.83	4.32	4.70	0.00	5.00	0.87	1.17	0.68	0.30
OK PRES 2020	5	66.94%	4.19	3.91	4.46	5.00	0.00	5.00	0.81	1.09	0.54	0.00
OK PRES 2016	5	69.32%	4.43	4.06	4.60	5.00	0.00	5.00	0.57	0.94	0.40	0.00

OH US SEN 2020	5	65.73%	4.07	3.83	4.38	5.00	0.00	5.00	0.93	1.17	0.62	0.00
OH US SEN 2016	5	73.35%	4.83	4.29	4.77	5.00	0.00	5.00	0.17	0.71	0.23	0.00
OH GOV 2018	5	56.27%	3.13	3.09	3.40	3.48	0.00	5.00	1.87	1.91	1.60	1.52
OR	6	42.85%	2.14	2.22	1.82	2.23	4.17	1.83	-0.31	-0.38	0.02	-0.40
OR PRES 2020	6	41.69%	2.00	2.09	1.61	1.79	5.00	1.00	-1.00	-1.09	-0.61	-0.79
OR PRES 2016	6	44.29%	2.31	2.35	2.01	3.15	5.00	1.00	-1.31	-1.35	-1.01	-2.15
OR US SEN 2020	6	40.87%	1.90	2.00	1.49	1.71	5.00	1.00	-0.90	-1.00	-0.49	-0.71
OR US SEN 2016	6	37.21%	1.47	1.66	1.03	1.01	3.00	3.00	1.53	1.34	1.97	1.99
OR GOV 2018	6	46.59%	2.59	2.60	2.39	2.56	4.00	2.00	-0.59	-0.60	-0.39	-0.56
OR GOV 2016	6	46.45%	2.57	2.59	2.37	3.15	3.00	3.00	0.43	0.41	0.63	-0.15
PA	17	47.11%	7.52	7.60	7.12	7.78	8.80	8.20	0.68	0.60	1.08	0.42
PA PRES 2020	17	49.41%	8.30	8.30	8.20	7.51	9.00	8.00	-0.30	-0.30	-0.20	0.49
PA PRES 2016	17	50.62%	8.71	8.71	8.82	8.28	8.00	9.00	0.29	0.29	0.18	0.72
PA US SEN 2018	17	43.34%	6.23	6.39	5.25	6.73	11.00	6.00	-0.23	-0.39	0.75	-0.73
PA US SEN 2016	17	50.88%	8.80	8.80	8.95	10.40	5.00	12.00	3.20	3.20	3.05	1.60
PA GOV 2018	17	41.32%	5.55	5.80	4.40	5.97	11.00	6.00	0.45	0.20	1.60	0.03
RI	2	38.88%	0.56	0.61	0.42	0.12	2.00	0.00	-0.56	-0.61	-0.42	-0.12
RI PRES 2020	2	39.40%	0.58	0.62	0.43	0.00	2.00	0.00	-0.58	-0.62	-0.43	0.00
RI PRES 2016	2	41.81%	0.67	0.70	0.54	0.31	2.00	0.00	-0.67	-0.70	-0.54	-0.31
RI US SEN 2020	2	33.40%	0.34	0.45	0.22	0.00	2.00	0.00	-0.34	-0.45	-0.22	0.00
RI US SEN 2018	2	38.40%	0.54	0.59	0.39	0.00	2.00	0.00	-0.54	-0.59	-0.39	0.00
RI GOV 2018	2	41.40%	0.66	0.69	0.52	0.31	2.00	0.00	-0.66	-0.69	-0.52	-0.31
SC	7	56.91%	4.47	4.39	4.85	5.41	1.00	6.00	1.53	1.61	1.15	0.59
SC PRES 2020	7	55.93%	4.33	4.28	4.70	5.35	1.00	6.00	1.67	1.72	1.30	0.65
SC PRES 2016	7	57.37%	4.53	4.46	4.96	5.27	1.00	6.00	1.47	1.54	1.04	0.73
SC US SEN 2020	7	55.22%	4.23	4.19	4.56	5.27	1.00	6.00	1.77	1.81	1.44	0.73
SC US SEN 2016	7	62.05%	5.19	4.98	5.70	5.91	1.00	6.00	0.81	1.02	0.30	0.09
SC GOV 2018	7	54.00%	4.06	4.04	4.33	5.25	1.00	6.00	1.94	1.96	1.67	0.75
SD	1						0.00	1.00				
TN	9	61.10%	6.50	6.26	7.10	6.83	1.10	7.90	1.40	1.64	0.80	1.07
TN PRES 2020	9	61.83%	6.63	6.38	7.29	6.83	1.00	8.00	1.37	1.62	0.71	1.17
TN PRES 2016	9	63.65%	6.96	6.62	7.59	6.83	1.00	8.00	1.04	1.38	0.41	1.17
TN US SEN 2020	9	63.86%	6.99	6.65	7.62	6.83	1.00	8.00	1.01	1.35	0.38	1.17
TN US SEN 2018	9	55.48%	5.49	5.43	5.93	6.83	1.00	7.50	2.01	2.57	2.07	0.67
TN GOV 2018	9	60.69%	6.42	6.22	7.08	6.83	1.50	8.00	1.58	1.28	0.42	1.17
TX	38	54.13%	22.14	21.98	23.57	20.34	13.60	24.40	2.26	2.42	0.83	4.06
TX PRES 2020	38	52.83%	21.15	21.09	22.20	18.03	13.00	25.00	3.85	3.91	2.80	6.97
TX PRES 2016	38	54.79%	22.64	22.47	24.33	21.94	14.00	24.00	1.36	1.53	-0.33	2.06
TX US SEN 2020	38	54.93%	22.75	22.56	24.48	21.77	13.00	25.00	2.25	2.44	0.52	3.23
TX US SEN 2018	38	51.31%	20.00	19.98	20.49	17.00	14.00	24.00	4.00	4.02	3.51	7.00
TX GOV 2018	38	56.77%	24.14	23.79	26.36	22.96	14.00	24.00	-0.14	0.21	-2.36	1.04

UT	4	66.40%	3.31	3.09	3.51	3.46	0.00	4.00	0.69	0.91	0.49	0.54
UT PRES 2020	4	60.69%	2.86	2.76	3.15	2.49	0.00	4.00	1.14	1.24	0.85	1.51
UT PRES 2016	4	62.28%	2.98	2.86	3.27	2.50	0.00	4.00	1.02	1.14	0.73	1.50
UT US SEN 2018	4	66.95%	3.36	3.13	3.57	3.94	0.00	4.00	0.64	0.87	0.43	0.06
UT US SEN 2016	4	71.28%	3.70	3.34	3.75	3.95	0.00	4.00	0.30	0.66	0.25	0.05
UT GOV 2020	4	67.48%	3.40	3.15	3.60	3.94	0.00	4.00	0.60	0.85	0.40	0.06
UT GOV 2016	4	69.74%	3.58	3.27	3.70	3.95	0.00	4.00	0.42	0.73	0.30	0.05
VT	1							0.50	0.50			
VA	11	44.72%	4.34	4.41	3.83	3.77	6.80	4.20	-0.14	-0.21	0.37	0.43
VA PRES 2020	11	44.85%	4.37	4.42	3.85	3.49	7.00	4.00	-0.37	-0.42	0.15	0.51
VA PRES 2016	11	47.42%	4.93	4.95	4.65	5.25	6.00	5.00	0.07	0.05	0.35	-0.25
VA US SEN 2020	11	44.00%	4.18	4.26	3.59	3.16	7.00	4.00	-0.18	-0.26	0.41	0.84
VA US SEN 2018	11	41.85%	3.71	3.85	2.99	3.08	7.00	4.00	0.29	0.15	1.01	0.92
VA GOV 2017	11	45.50%	4.51	4.55	4.05	3.89	7.00	4.00	-0.51	-0.55	-0.05	0.11
WA	10	42.10%	3.42	3.55	2.80	2.74	6.67	3.33	-0.09	-0.22	0.54	0.59
WA PRES 2020	10	40.07%	3.01	3.21	2.30	2.59	7.00	3.00	-0.01	-0.21	0.70	0.41
WA PRES 2016	10	41.25%	3.25	3.40	2.57	2.69	7.00	3.00	-0.25	-0.40	0.43	0.31
WA US SEN 2018	10	41.60%	3.32	3.46	2.65	2.56	7.00	3.00	-0.32	-0.46	0.35	0.44
WA US SEN 2016	10	40.90%	3.18	3.35	2.49	2.37	7.00	3.00	-0.18	-0.35	0.51	0.63
WA GOV 2020	10	43.30%	3.66	3.75	3.08	2.69	6.00	4.00	0.34	0.25	0.92	1.31
WA GOV 2016	10	45.50%	4.10	4.14	3.68	3.57	6.00	4.00	-0.10	-0.14	0.32	0.43
WV	2	72.16%	1.89	1.69	1.89	2.00	0.00	2.00	0.11	0.31	0.11	0.00
WV PRES 2016	2	72.16%	1.89	1.69	1.89	2.00	0.00	2.00	0.11	0.31	0.11	0.00
WI	8	49.20%	3.87	3.88	3.82	4.57	2.60	5.40	1.53	1.52	1.58	0.83
WI PRES 2020	8	49.68%	3.95	3.95	3.92	5.02	2.00	6.00	2.05	2.05	2.08	0.98
WI PRES 2016	8	50.53%	4.08	4.08	4.13	5.15	2.00	6.00	1.92	1.92	1.87	0.85
WI US SEN 2018	8	44.60%	3.14	3.18	2.74	3.21	4.00	4.00	0.86	0.82	1.26	0.79
WI US SEN 2016	8	51.75%	4.28	4.28	4.42	4.87	2.00	6.00	1.72	1.72	1.58	1.13
WI GOV 2018	8	49.44%	3.91	3.91	3.87	4.59	3.00	5.00	1.09	1.09	1.13	0.41
WY	1							0.00	1.00			

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